13 Equivalence of Sets

90. Show that $\mathbb{N}$ (the set of natural numbers) and $\mathbb{Z}$ (the set of integers) are two equinumerous sets.

91. Find a bijection between the interval $(0, 1)$ and the interval (a, b).

92. **(exam, February 2021)** First, show that there exists an injective function $f : \mathbb{Z} \to \mathbb{N}$. Then prove that $|\mathbb{Z}| = |\mathbb{N}|$.

93. **(2nd colloquium, January 2022)** Let $2\mathbb{N}$ be the set of all even numbers (i.e., $2\mathbb{N} = \{2n \mid n \in \mathbb{N}\}$). First, show that there exists an injective function $f : \mathbb{Z} \to 2\mathbb{N}$. Then prove that $|\mathbb{Z}| = |2\mathbb{N}|$.

94. **(2nd colloquium, January 2021)** Show that the intervals $[2, \infty)$ and $[-3, 3)$ are equinumerous.

95. **(exam, June 2021)** Show that the intervals $(0, 2)$ and $[1, 3)$ are equinumerous.

96. First, show that there exists an injective function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{N} \times \mathbb{N}$. Then prove that $|\mathbb{Z} \times \mathbb{Z}| = |\mathbb{N} \times \mathbb{N}|$.

97. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{Z}$ be the set of integers. Justify whether the statement

$$|\mathbb{Z} \times \mathbb{Z}| = |\mathbb{N}|$$

holds for the sets $\mathbb{Z} \times \mathbb{Z}$ and $\mathbb{N}$.

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
The reader can find all solutions to the given problems on the same page.